JHEP HYPER VERSION

Received: *April 26, 2001,* Accepted: *June 12, 2001*

M-theory on manifolds of G_2 holonomy and type-IIA orientifolds

Shamit Kachru[ab] **and John McGreevy**[a]

[a]*Department of Physics and SLAC, Stanford University*
Stanford, CA 94305/94309, USA
[b]*Institute for Theoretical Physics, University of California*
Santa Barbara, CA 93106, USA
E-mail: skachru@stanford.edu, mcgreevy@stanford.edu

Abstract: We demonstrate that M-theory compactifications on 7-manifolds of G_2 holonomy, which yield 4d $\mathcal{N} = 1$ supersymmetric systems, often admit at special loci in their moduli space a description as type-IIA orientifolds. In this way, we are able to find new dualities of special IIA orientifolds, including dualities which relate orientifolds of IIA strings on manifolds of different topology with different numbers of wrapped D-branes. We also discuss models which incorporate, in a natural way, compact embeddings of gauge theory/gravity dualities similar to those studied in the recent work of Atiyah, Maldacena and Vafa.

Keywords: M-Theory, String Duality, Superstring Vacua.

JHEP06(2001)027

Contents

JHEP06(2001)027

1. Introduction

Compactifications of M-theory and string theory down to 4d $\mathcal{N} = 1$ supersymmetry are of obvious interest. The reduced supersymmetry is probably necessary for any contact with real world physics. It also allows for richer phenomena than extended supersymmetry and so provides a nice playground for theorists.

Generic methods of constructing such models include compactifying the heterotic string on Calabi-Yau threefolds, F-theory on Calabi-Yau fourfolds, and M-theory on 7-manifolds of G_2 holonomy. Although many basic facts about all of these classes of compactifications remain mysterious, perhaps the least is known about the last class, since at least the others are amenable to attack using techniques of complex geometry.

A large class of compact 7-manifolds with G_2 holonomy was constructed by Joyce [1, 2]. In this note, we make the simple observation that M-theory compactified on many of these spaces admits, at special loci in its moduli space, a description as an orientifold of type-IIA string theory compactified on a Calabi-Yau threefold.[1] This is reminiscent of the fact that F-theory models can be reformulated, at special loci in their moduli space, as type-IIB orientifolds [4, 5].

There are several different reasons this observation can be useful. On the one hand, the orientifolds we discuss have a rather simple, solvable structure, and so provide a very concrete handle on these models at some special points in their moduli space. On the other hand, as we will show, a given G_2 space can admit *different* type-IIA orientifold limits. Thus, by studying limit points in the moduli space of G_2 compactifications, we learn about non-perturbative dualities of IIA compactifications with $\mathcal{N} = 1$ supersymmetry. In particular, we exhibit an example where orientifolds of type IIA on Calabi-Yau spaces of different topology (and with different numbers of D-branes and orientifold planes) are dual to each other. On yet a third hand, our construction "globalizes" interesting gauge theory/gravity dualities similar to those encapsulated in the local models of [6] and [7].

In section 2, we introduce the G_2 manifold X which will be our focus in part of this note. In section 3, we show that various limit points in the moduli space of M-theory on X are well described by IIA orientifolds. This observation allows us to find non-perturbative duality symmetries of these orientifolds. In section 4 we make some remarks about the extent to which our analysis generalizes to other G_2 spaces, and also provide a simple proof that a large class of IIA orientifolds should have an "M-theory lift" to G_2 compactifications. In section 5 we explain how gauge theory/gravity dualities analogous to those discussed in [6, 7] naturally arise in simple examples of compact G_2 manifolds and the related IIA orientifolds. We conclude in section 6 by mentioning some interesting directions for further study.

Several papers analyzing various related aspects of M-theory on G_2 manifolds have appeared recently. Dual descriptions of $\mathcal{N} = 1$ gauge theories using such spaces have been discussed in [6]–[11], while [12] discusses a general relationship between certain classes of wrapped branes and geometries with exceptional holonomy. Earlier work on this subject appears in [13]. Phase transitions between topologically distinct G_2 compactifications were described in [14].

2. The manifold X

A basic example of a compact 7-manifold of G_2 holonomy is the manifold X considered by Joyce in [1]. It is constructed as a toroidal orbifold. Let $x_1, \ldots, x_7$ parametrize a square T^7 which is a product of seven circles of radii $r_1, \ldots, r_7$. De-

[1] We ignore the possibility of a membrane instanton generated superpotential [3] in most of our discussion. If such a potential exists, it would provide a potential barrier between the large-volume M-theory and perturbative type-IIA limits. Our successful comparison of these limits in many cases suggests that either such a potential is absent, or the interpolation over the potential barrier is nevertheless physically meaningful.

fine X as the (desingularization of the) quotient of this T^7 by the $\mathbb{Z}_2^3$ group with generators

$$\alpha(x_i) = (-x_1, -x_2, -x_3, -x_4, x_5, x_6, x_7) \tag{2.1}$$

$$\beta(x_i) = \left(-x_1, \frac{1}{2} - x_2, x_3, x_4, -x_5, -x_6, x_7\right) \tag{2.2}$$

$$\gamma(x_i) = \left(\frac{1}{2} - x_1, x_2, \frac{1}{2} - x_3, x_4, -x_5, x_6, -x_7\right), \tag{2.3}$$

where $1/2$ denotes a shift of order 2 around the circle. Then as demonstrated in [1], X has betti numbers $b_2(X) = 12$, $b_3(X) = 43$. Therefore, M-theory compactification on X gives rise to a 4d $\mathcal{N} = 1$ supersymmetric low-energy theory with (generically) 12 abelian vector multiplets and 43 chiral multiplet moduli.

It may be useful to review the origin of the various cohomology classes on X here. None of the two-forms and seven of the three-forms on T^7 are invariant under the action of $\langle\alpha, \beta, \gamma\rangle$.[2] In addition, each of the generators fixes 16 T^3s on T^7; however e.g. the 16 T^3s fixed by α are identified by the group $\langle\beta, \gamma\rangle$ to yield 4 on the quotient X, and the fixed tori of β and γ undergo a similar fate. The local form of the singularities at the fixed T^3s is $\mathbb{R}^4/\mathbb{Z}_2 \times T^3$, and resolving each of these yields a two-form and three three-forms. Since there are 12 such fixed tori on X, after desingularizing one has the stated betti numbers.

3. Orientifold limits of X

In this section, we demonstrate that X has several different IIA orientifold limits in its moduli space. This in particular tells us that the different orientifolds are related to one another by various dualities.

3.1 Orientifold A

We start by viewing x_7 as the "M-theory circle," or the eleventh dimension. Then in the limit of small r_7, we should be able to get an effective IIA description of M-theory on X. Denote by α^* the action of α restricted to the T^6 with coordinates $x_1, \ldots, x_6$. Then since α and β don't act on the M-theory circle anyway, in the limit of small x_7 they simply induce identifications on the T^6 visible to the type-IIA string. It is therefore propagating on the Calabi-Yau space $N = T^6/\langle\alpha^*, \beta^*\rangle$.

However, γ also acts on the M-theory circle. Using the results of [15], it follows that the action of γ (inversion of the M-theory circle and three other coordinates) is mapped in the IIA theory to $w = (-1)^{F_L}\Omega\gamma^*$. Thus, IIA string theory on the orientifold of N by w should govern M-theory in the limit of small radius for x_7. Let us call this model orientifold A.

[2] $\langle\cdots\rangle$ denotes "the group generated by $\cdots$."

To check this conjecture, let us try to match up the counting of fields. N has hodge numbers $h^{1,1} = 19$, $h^{2,1} = 19$. So IIA string theory on N yields an $\mathcal{N} = 2$ supersymmetric 4d theory with 20 hypermultiplets (including the dilaton) and 19 vector multiplets. Now, projecting by w has the following effect (see e.g. [16]). Each of the 20 hypermultiplets is projected down to a chiral multiplet. The vector multiplets (which came from the Kähler moduli) are more subtle: those which come from (1,1) forms *invariant* under w give rise to $\mathcal{N} = 1$ vector multiplets, while those which are *anti-invariant* under w give rise to chiral multiplets. It is easy to convince oneself that the untwisted (1,1) forms on N are anti-invariant, while the 16 twisted (1,1) forms split into $\pm$ eigenspaces of equal size. Therefore, the Kähler moduli contribute 11 chiral multiplets and 8 vector multiplets.

This accounts for 31 chiral multiplets and 8 abelian vectors so far. However, we must also take into account the fixed points of the w action. γ^* acts with 8 fixed loci on T^6. Identification by $\langle\beta, \gamma\rangle$ reduces this to 2 fixed loci; a neighborhood of each in the threefold is of the form $\mathbb{R}^3/\mathbb{Z}_2 \times T^3$. Therefore, there are orientifold six-planes wrapping each of these T^3s (as in [15]).

By the normal tadpole cancellation considerations, we must introduce 2 D6-branes for each O6-plane. Hence, we introduce a total of 4 D6-branes wrapping T^3s in this model. Each of the D6-branes comes with a U(1) vector multiplet and 3 chiral multiplet moduli (coming from the Wilson lines on the T^3, together with the moduli of the three-cycle in N). So the D6-branes contribute a total of 12 chiral multiplet moduli and 4 vector multiplets to the low-energy theory.

Totalling up the spectrum, we find that orientifold A (at generic points in its moduli space) has 43 chiral multiplet moduli and 12 vector multiplets, just as it must to match the spectrum of M-theory on X. At special points in moduli space when the D6-branes coincide, one achieves enhanced gauge symmetries, which come from geometrical singularities in the M-theory picture [15].

3.2 Orientifold B

It is of course also possible to view other circles as the M-theory circle. For instance, we could take x_4 to be the M-theory circle. However, repeating the same logic as in section 3.1, we would find that we again arrive (in the small r_4 limit) at an orientifold (which we could call orientifold B) of type-IIA string theory on the Calabi-Yau orbifold with hodge numbers $h^{1,1} = 19$, $h^{2,1} = 19$, and we again have to introduce the same numbers of D6-branes.

The role of the dilaton in orientifold A is played by a geometrical modulus in orientifold B, and vice-versa. However, they are really compactifications on the same target space. This means that the IIA theory on the orientifold of N by w, discussed in section 3.1, has a sort of S-T exchange symmetry, where S is the dilaton chiral multiplet and T is the chiral multiplet containing the radius of x_4. This way of seeing the S-T exchange symmetry of these orientifold models is analogous to the way that the S-T exchange symmetry [17] of the main heterotic string examples in [18] can be understood as arising from the existence of multiple $K3$ fibrations in the type-II-dual Calabi-Yau compactifications [19, 20].

In fact, this model enjoys more symmetry than just a single S-T duality; one could equivalently consider the x_6 circle to be the M-theory circle, with the same results, yielding a sort of S-T-U triality symmetry.

3.3 Orientifold C

A more interesting possibility is to interpret x_5 as the M-theory circle. Then acting on the T^6 coordinates $x_1, x_2, x_3, x_4, x_6, x_7$, we have

$$\alpha(x_i) = (-x_1, -x_2, -x_3, -x_4, x_6, x_7) \tag{3.1}$$

$$\beta\gamma(x_i) = \left(x_1 + \frac{1}{2}, \frac{1}{2} - x_2, -x_3, x_4, -x_6, -x_7\right) \tag{3.2}$$

The manifold $N' = T^6/\langle\alpha, \beta\gamma\rangle$ is then a Calabi-Yau threefold with hodge numbers $h^{1,1} = 11, h^{2,1} = 11$. In particular, it is topologically distinct from the threefold N which appeared in section 3.1 and section 3.2.

Define u to be the composition of $(-1)^{F_L}\Omega$ with the action of γ on the T^6 coordinates. Then in the limit of small r_5, M-theory on X should be well described by IIA theory on the orientifold of N' by u, which we will call orientifold C.

Let's check that the spectrum matches our expectations. v, the composition of $(-1)^{F_L}\Omega$ with the action of β on the T^6 coordinates, arises upon composing u with elements of the orbifold group. Both u and v act with fixed loci on N'. Each has 8 fixed T^3s in the T^6, which descend to 2 fixed T^3s in the orbifold N'. Therefore, one has to introduce four O6-planes, and 8 wrapped D6-branes are required to cancel the RR tadpoles. These give rise to 8 abelian vector multiplets and 24 chiral multiplets, at generic points in moduli space.

The projection of the spectrum of IIA on N' can be done as before. Once again, half of the 8 twisted (1,1) forms are invariant under the orientifold action, while the other half (and the untwisted (1,1) forms) are anti-invariant. So we get 4 vectors and 7 chirals from the (1,1) forms; adding in the 12 chirals descending from the $\mathcal{N} = 2$ hypermultiplets, we indeed find a total of 43 chiral multiplets and 12 vectors.

In this orientifold C picture, the radii r_4, r_6, r_7 which are related (up to triality) to the dilaton in the pictures of section 3.1 and section 3.2 are all geometrical moduli of the IIA compactification, while r_5 (which is geometrized in orientifolds A,B) is playing the role of the dilaton. This gives an example of a strong/weak duality between IIA orientifolds of topologically distinct Calabi-Yau spaces, with different numbers of space-filling D-branes and orientifold planes.

JHEP06(2001)027

4. Generalization to other models

In this section, we generalize our results in two directions. We first show that a large class of G_2 spaces should similarly have orientifold limits. We then take the opposite approach, and prove that a wide class of IIA orientifolds have an M-theory lift to G_2 compactifications.

4.1 Other classes of G_2 manifolds

Beyond the toroidal orbifold constructions of Joyce, there are other methods of constructing G_2 holonomy spaces which are amenable to an orientifold interpretation.

Barely G_2 manifolds. Harvey and Moore defined "barely G_2 manifolds" as quotients of the form $X = (Y \times S^1)/\mathbb{Z}_2$, where Y is a Calabi-Yau threefold and the $\mathbb{Z}_2$ action is a composition of a freely acting antiholomorphic involution σ on Y with inversion on the circle factor x_7 [3]. These are of course a special case of a more general construction which should arise when σ has fixed points [1, 2].

For the barely G_2 spaces, it turns out that

$$H^3(X) = H^2(Y)^- + H^3(Y)^+ \tag{4.1}$$

$$H^2(X) = H^2(Y)^+ \,, \tag{4.2}$$

where $\pm$ refer to eigenvalues under the action of σ on Y. For simple examples which come from hypersurfaces in toric varieties, one simply keeps the complex structure deformations which preserve the real structure (i.e. defining equations with real coefficients), so $H^3(Y)$ has $\pm$ eigenspaces of equal dimension. For such examples, we find $n_C = h^{1,1}(Y)^- + h^{2,1}(Y) + 1$ chiral multiplets and $n_V = h^{1,1}(Y)^+$ vector multiplets in M-theory on X.

As one shrinks the radius r_7 of the S^1, one should obtain a IIA description. Indeed, since the $\mathbb{Z}_2$ above acts with an inversion on x_7, we should expect that the orientifold of IIA on Y by $(-1)^{F_L}\Omega$ composed with σ arises in this limit. It follows from the general considerations of [16] (as discussed in section 3) that the spectrum of this type-IIA orientifold agrees with the M-theory spectrum.

Cases with fixed points. It is attractive to speculate about generalizations of the previous case to cases where σ acts on Y with fixed points. On general grounds, the fixed point locus $\Sigma \subset Y$ will be a special Lagrangian (sL) three-cycle (or several, in which case one should repeat the discussion below for each component). It is not known in generality how to resolve the singularities in this case to obtain a smooth metric of G_2 holonomy.

However, the existence of an orientifold limit leads to a very natural conjecture. Shrinking the x_7 circle again, we find a IIA model which should have an O6-plane and two D6-branes wrapping Σ. For Σ special lagrangian, a D6-brane wrapping Σ

gives rise to a single $\mathcal{N}=1$ vector multiplet and $b_1(\Sigma)$ chiral multiplets in space-time. Therefore, we expect that there will be 2 vectors and $2b_1(\Sigma)$ chiral multiplets associated with the D6-branes in this limit. When the D6-branes are coincident, the model has enhanced gauge symmetry (which shows up in the M-theory as the singularity of the G_2 space related to the fixed points of σ). For $b_1(\Sigma)>0$, one can move in the D6-brane moduli space to remove the enhanced gauge symmetry. It is then attractive to conjecture that in the M-theory picture, $b_1(\Sigma)>0$ is a condition that allows the singularities of this class of G_2 orbifolds to be repaired, and that furthermore resolving the singularity gives rise to precisely two elements of $b_2(X)$ and $2b_1(\Sigma)$ elements of $b_3(X)$.[3]

4.2 "All" orientifolds of type-IIA on CY have a G_2 limit

Suppose we have a IIA orientifold which gives rise to a four dimensional $\mathcal{N}=1$ supersymmetric theory. For simplicity, let us first restrict ourselves to orientifolds of tori. The orbifold part of the orientifold group must have (at most) SU(3) holonomy, to preserve (at least) 4d $\mathcal{N}=2$ supersymmetry.[4] Let us assume we are in the most generic case, so that it preserves precisely $\mathcal{N}=2$ supersymmetry. Denote the full orientifold group by

$$\mathcal{G}=\Gamma_1\times(-1)^{F_L}\Omega\Gamma_2\,. \tag{4.3}$$

The $(-1)^{F_L}$ is present because we choose, as in [15], a convention where reflection on three circles must be accompanied by a $(-1)^{F_L}$ to preserve supersymmetry in the IIA theory, and we will show momentarily that all elements of Γ_2 must reflect precisely three circles of the T^6. With these assumptions, T^6/Γ_1 alone is Calabi-Yau, and so has a holomorphic three-form $\Omega^{(3,0)}$ and a Kähler form J.

Now, consider the $(-1)^{F_L}\Omega\Gamma_2$ part of the group. Any element $(-1)^{F_L}\Omega g_2$ of this part must have g_2 reversing the orientation of the 6d target, or it cannot be a symmetry of the IIA theory. So we know a few things about the g_2 action:

(i) g_2 maps J to $-J$ (orientation reversal) and

(ii) g_2 maps $\Omega^{(3,0)}$ to $\overline{\Omega}^{(0,3)}$. Notice that this implies that g_2 reflects precisely three circles of the T^6/Γ_1, as required above.

In (ii), we are using the fact that to preserve one supersymmetry, some linear combination of the killing spinors must be preserved. This means that g_2 either preserves the holomorphic and anti-holomorphic three-form individually, or at least preserves a linear combination. But since $\Omega^{(3,0)}\wedge\overline{\Omega}^{(0,3)}\sim J\wedge J\wedge J$, by (i) above g_2 must permute the two. One might worry that g_2 could act with a phase in relating

[3] This could be related to Condition 4.3.1 in [2], which was stated without proof to be an important condition in resolving singularities of this sort.

[4] This is because there are no geometric compactifications of IIA down to 4d which preserve precisely 4d $\mathcal{N}=1$ supersymmetry.

$\Omega^{(3,0)}$ to its conjugate; but all $g_2 \subset \Gamma_2$ which exchange $\Omega^{(3,0)}$ and its conjugate would have to have the same phase to preserve $\mathcal{N} = 1$ supersymmetry. It can then be redefined to 1 by a phase rotation of $\Omega^{(3,0)}$.

To proceed, we add a seventh M-theory circle x_7. Define the new group $\tilde{\mathcal{G}}$, which acts on T^7, as follows: take each element of $\mathcal{G}$ and replace $(-1)^{F_L}\Omega$ anywhere it appears with inversion of the x_7 coordinate (while elements which don't include a $(-1)^{F_L}\Omega$ act trivially on the x_7 coordinate). This will not change anything about elements of Γ_1 (since the minus sign on x_7 will cancel in the product of two $(-1)^{F_L}\Omega\Gamma_2$ elements). However, under assumptions (i) and (ii) above, the three-form

$$\Phi = J \wedge dx_7 + \mathrm{Re}[\Omega^{(3,0)}] \tag{4.4}$$

is preserved by the whole (now orbifold) group $\tilde{\mathcal{G}}$ acting on T^7. This form is preserved by a G_2 subgroup of $\mathrm{GL}(7,\mathbb{R})$ [2]. This is sufficient to prove that the resulting manifold is a G_2 space.

It is clear that this argument is more generically applicable to supersymmetric models which are not toroidal orientifolds. One could replace the T^6 in the IIA theory with any manifold M, use the fact that M/Γ_1 should be Calabi-Yau to preserve supersymmetry in the IIA theory, and apply the same logic.

5. The case of the disappearing orientifold

Recent work has made it clear that gauge dynamics on wrapped D6-branes (or arising from singular M-theory geometries) can often be encoded by smooth geometries in a "dual" gravity description [6]. The gauge dynamics is then encoded in appropriate RR-fluxes, or in changes of the behavior of the M-theory three-form C field, which (suitably interpreted) capture the low-energy physics of the gauge theory. In this section, we discuss examples of this phenomenon which arise in string/M-theory compactifications in a natural way.

The most obvious source of consistent compact models with wrapped D6-branes is the Calabi-Yau orientifolds discussed here. The components of the orientifold fixed locus provide sL three-cycles Σ, which are wrapped by orientifold planes and D6-branes. In fact, examples of sL cycles Σ which arise in this way were studied in [21, 22] precisely with the motivation of understanding the dynamics on the worldvolumes of such wrapped D6-branes.

One interesting fact (which had perplexed some of the authors of [21, 22] for some time) is that it is possible for the fixed locus of an anti-holomorphic involution to disappear as the complex structure of the Calabi-Yau varies; and the relevant complex structure moduli survive in the orientifold models. This fact was used in [22] to identify D6-branes on such real slices as mirror to D5-branes on vanishing holomorphic curves. However, it raises the question: if one continues past the point in moduli space where the fixed locus disappears (so there is no orientifold plane, and no need to introduce D6-branes), where has the information about the gauge theory on the D6-branes gone? The gauge theory/gravity dualities relevant to this situation were studied in [6, 7], and provide the answer to this question.

Let us illustrate this with a simple example. The easiest examples discussed in [22] basically involve a sL three-cycle which is the fixed locus of a real involution and which collapses at a conifold singularity. So locally, the geometry of the compact Calabi-Yau M looks like

$$z_1^2 + z_2^2 + z_3^2 + z_4^2 = \mu \,, \tag{5.1}$$

where μ is chosen to be a *positive* real parameter. Then under the involution

$$\mathcal{I} : z_i \to \overline{z}_i \tag{5.2}$$

the fixed point locus Σ^+ is the three-sphere

$$\Sigma^+ : \ \sum_{i=1}^{4} x_i^2 = \mu \,, \tag{5.3}$$

where $z_i = x_i + i y_i$.

We can embed this situation in a G_2 manifold as in section 4.1, where the G_2 manifold X is of the form $(M \times S^1)/\sigma$. The $\mathbb{Z}_2$ symmetry σ acts by $\mathcal{I}$ combined with inversion on the M-theory circle, $x_7 \to -x_7$. Then for $\mu > 0$, the fixed point loci of σ, which consist of copies of Σ^+ at $x_7 = 0, 1/2$, are actually S^3s of A_1 singularities in X. This gives rise in M-theory on X to two 4d, $\mathcal{N} = 1$ pure SU(2) gauge theories (with equal gauge couplings).

Now, consider taking μ through 0. At $\mu \to 0$ there are collapsing associative three-cycles in X, and hence membrane instanton effects are expected to be large [3]. However, the sizes of the S^3s come paired in chiral multiplets with periods of the three-form C field over the S^3s, and for generic values of this phase, there is no singularity in the physics – singularities in $\mathcal{N} = 1$ moduli spaces happen at complex codimension one. Therefore, one can smoothly (in the physical sense) continue from $\mu > 0$ to $\mu < 0$. This raises a puzzle: the SU(2) gauge groups present for $\mu > 0$ have now disappeared, since the $\mathbb{Z}_2$ symmetry σ acts on X without fixed points for $\mu < 0$. However, the information about the gauge theory must be encoded somehow in the $\mu < 0$ geometry.

The basic point is as in [6]. For $\mu < 0$, one can still look for a homologically nontrivial three-sphere which membrane instantons can wrap. For instance, consider the locus of pure imaginary z_i, still at $x_7 = 0, 1/2$. This is given by a three-sphere

$$\sum_{i=1}^{4} y_i^2 = -\mu \,, \tag{5.4}$$

which is orbifolded by the freely-acting $\mathbb{Z}_2$ symmetry $y_i \to -y_i$. Call the resulting $\mathbb{RP}^3$ Σ^-. It turns out that $\Sigma^-(-\mu)$ has exactly half the volume of $\Sigma^+(\mu)$, due to the orbifolding. These $\mathbb{RP}^3$s are associative three-cycles in X for $\mu < 0$. However, as in [6], changing the period of the C field on $\Sigma^+(\mu)$ by 2π, which is physically meaningless, corresponds to changing it by π on $\Sigma^-(-\mu)$, due to the smaller volume. This ambiguity in the choice of phase for $\mu < 0$ corresponds to the vacuum degeneracy due to the gaugino condensate in the gauge theory.[5]

In the IIA picture, with x_7 taken as the M-theory circle, this becomes an example where an orientifold plane and two D6-branes, present for $\mu > 0$, disappear as μ passes through 0. This system has an SO(4) gauge symmetry, and should give rise to multiple vacua after gaugino condensation, in agreement with the M-theory picture above. The phase ambiguity detected by membrane instantons in M-theory is detected by D2-brane instantons in the string theory picture. This is in accord with our gauge theory intuition, since D2-branes are the instantons of the D6-brane gauge theory in the phase where the D6-branes exist [23]. The fact that $\Sigma^-(-\mu)$ has half the volume of $\Sigma^+(\mu)$ then becomes the familiar fact that the superpotential from a gaugino condensate in $\mathcal{N} = 1$ SU(2) gauge theory looks like a "half-instanton effect".

More precisely, once we have compactified this setup, the superpotential we are discussing destabilizes the closed-string modulus μ (which is a parameter in the non-compact case). In our discussion here, we are imagining that we can hold μ fixed at various values, which is reasonable as long as the scale generated by the superpotential is parametrically smaller than the string/Planck scale. This is true for large enough $|\mu|$.

It is clear that the other examples of [22], which involve sL three-cycles Σ with $b_1(\Sigma) > 0$, could also be lifted in this way to find examples of M-theory "dualities" in gauge theories with adjoint matter. Some examples of this have already appeared in [9].

6. Discussion

Little is known about M-theory compactification on spaces of G_2 holonomy. Naive extrapolation of the kinds of results that exist so far suggests that further study of the relationship between IIA orientifolds and M-theory compactifications could yield:

1. A large class of examples of non-perturbative dualities between orientifolds of type-II compactifications on Calabi-Yau spaces of different topologies, with different numbers of space-filling D-branes.

[5] Notice that since the two associative S^3s at $x_7 = 0, 1/2$ are in the same homology class, their volumes (and the periods of the C-field) are the same. So although there are two SU(2)s, the choice of phase in the two gaugino condensates is related – there is only a single $\mathbb{Z}_2$ ambiguity. This carries over to the IIA picture as well.

2. New gauge theory/gravity dualities along the lines of [6], in a compact context (i.e., coupled to 4d gravity).

3. Connections between the study of disc instanton effects in type-II compactifications with branes (see e.g. [21, 22, 24, 25]) and membrane instanton effects in M-theory [3]. The real involution of the CY pairs holomorphic discs [22] even when the sL three-cycle on which they end is deformed away from the real slice. In this way, pairs of discs times the M-theory circle form closed orbifold-invariant three-manifolds which membrane instantons can wrap. Similarly, $\mathbb{RP}^2$ worldsheets with their crosscap on the real slice lift to orbifolds of membrane instantons on the M-theory circle times the covering sphere of the $\mathbb{RP}^2$ [7].

4. A good understanding of the new physics which arises at singularities of M-theory on spaces of G_2 holonomy (some examples of this were discussed in [10]). It would be particularly interesting to find various singularities which correspond to chiral gauge theories. Perhaps these would provide a useful tool for the further exploration of chirality changing phase transitions [26].

5. A new window into type-I compactifications. The type-IIA orientifolds studied here are T-dual to type-I string compactifications (roughly speaking, by T-duality on the T^3 fibers [27] of the Calabi-Yau space which is being orientifolded). Therefore, any insights gained about these models through their M-theory interpretation will carry over to the study of certain type-I theories.

Acknowledgments

We would like to thank J. Polchinski and E. Silverstein for helpful discussions, and B. Acharya for pointing out a minor error in section 5 of the first version of this note. This work was supported in part by NSF grant PHY-95-14797 and by the DOE under contract DOE-AC03-76SF00098. S.K. enjoyed the hospitality of the ITP Santa Barbara "M-theory" program while performing the work reported here, and was supported by the National Science Foundation under grant number PHY-99-07949. The work of S.K. was supported in part by a David and Lucile Packard Foundation Fellowship for Science and Engineering and an Alfred P. Sloan Foundation Fellowship. The work of J. M. was supported in part by the Department of Defense NDSEG Fellowship program.

References

[1] D.D. Joyce, *Compact riemannian 7-manifolds with G_2 holonomy, 1*, *J. Diff. Geom.* **43** (1996) 291.

[2] D.D. Joyce, *Compact riemannian 7-manifolds with G_2 holonomy, 2*, *J. Diff. Geom.* **43** (1996) 329.

[3] J.A. Harvey and G. Moore, *Superpotentials and membrane instantons*, hep-th/9907026.

[4] A. Sen, *F-theory and orientifolds*, *Nucl. Phys.* **B 475** (1996) 562 [hep-th/9605150]; *Orientifold limit of F-theory vacua*, *Phys. Rev.* **D 55** (1997) 7345 [hep-th/9702165].

[5] R. Gopakumar and S. Mukhi, *Orbifold and orientifold compactifications of F-theory and M-theory to six and four dimensions*, *Nucl. Phys.* **B 479** (1996) 260 [hep-th/9607057].

[6] M. Atiyah, J. Maldacena and C. Vafa, *An M-theory flop as a large-N duality*, hep-th/0011256.

[7] S. Sinha and C. Vafa, SO *and* Sp *Chern-Simons at large-N*, hep-th/0012136.

[8] B.S. Acharya, *On realising $N = 1$ super Yang-Mills in M-theory*, hep-th/0011089; *Confining strings from G_2-holonomy spacetimes*, hep-th/0101206; B. Acharya and C. Vafa, *On domain walls of $N = 1$ supersymmetric Yang-Mills in four dimensions*, hep-th/0103011.

[9] F. Cachazo, K. Intriligator and C. Vafa, *A large-N duality via a geometric transition*, *Nucl. Phys.* **B 603** (2001) 3 [hep-th/0103067].

[10] E. Witten, *M-theory dynamics on manifolds of G_2 holonomy*, talk presented March 3, 2001 at *Heterotic dreams and asymptotic visions*, the 60th birthday celebration for David Gross, ITP, Santa Barbara.

[11] J.D. Edelstein and C. Nunez, *D6 branes and M-theory geometrical transitions from gauged supergravity*, *J. High Energy Phys.* **04** (2001) 028 [hep-th/0103167].

[12] J. Gomis, *D-branes, holonomy and M-theory*, hep-th/0103115.

[13] B.S. Acharya, *M-theory, Joyce orbifolds and super Yang-Mills*, *Adv. Theor. Math. Phys.* **3** (1999) 227 [hep-th/9812205].

[14] H. Partouche and B. Pioline, *Rolling among G_2 vacua*, *J. High Energy Phys.* **03** (2001) 005 [hep-th/0011130].

[15] A. Sen, *A note on enhanced gauge symmetries in M and string theory*, *J. High Energy Phys.* **09** (1997) 001 [hep-th/9707123]

[16] C. Vafa and E. Witten, *Dual string pairs with $\mathcal{N} = 1$ and $\mathcal{N} = 2$ supersymmetry in four dimensions*, *Nucl. Phys.* **46** *(Proc. Suppl.)* (1996) 225 [hep-th/9507050].

[17] A. Klemm, W. Lerche and P. Mayr, *K3 fibrations and heterotic-type-II string duality*, *Phys. Lett.* **B 357** (1995) 313 [hep-th/9506112].

[18] S. Kachru and C. Vafa, *Exact results for $\mathcal{N}=2$ compactifications of heterotic strings*, *Nucl. Phys.* **B 450** (1995) 69 [hep-th/9505105].

[19] P.S. Aspinwall and M. Gross, *Heterotic-heterotic string duality and multiple K3 fibrations*, *Phys. Lett.* **B 382** (1996) 81 [hep-th/9602118].

[20] D.R. Morrison and C. Vafa, *Compactifications of F-theory on Calabi-Yau threefolds, 1*, *Nucl. Phys.* **B 473** (1996) 74 [hep-th/9602114].

[21] S. Kachru, S. Katz, A.E. Lawrence and J. McGreevy, *Open string instantons and superpotentials*, *Phys. Rev.* **D 62** (2000) 026001 [hep-th/9912151].

[22] S. Kachru, S. Katz, A.E. Lawrence and J. McGreevy, *Mirror symmetry for open strings*, *Phys. Rev.* **D 62** (2000) 126005 [hep-th/0006047].

[23] M.R. Douglas, *Branes within branes*, hep-th/9512077.

[24] H. Ooguri and C. Vafa, *Knot invariants and topological strings*, *Nucl. Phys.* **B 577** (2000) 419 [hep-th/9912123].

[25] M. Aganagic and C. Vafa, *Mirror symmetry, D-branes and counting holomorphic discs*, hep-th/0012041.

[26] S. Kachru and E. Silverstein, *Chirality-changing phase transitions in 4d string vacua*, *Nucl. Phys.* **B 504** (1997) 272 [hep-th/9704185];
B.A. Ovrut, T. Pantev and J. Park, *Small instanton transitions in heterotic M-theory*, *J. High Energy Phys.* **05** (2000) 045 [hep-th/0001133].

[27] A. Strominger, S.-T. Yau and E. Zaslow, *Mirror symmetry is T-duality*, *Nucl. Phys.* **B 479** (1996) 243 [hep-th/9606040].